

Inverse Hyperbolic Functions

Definitions

$$\text{arcsinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\text{arccosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$\text{arctanh}(x) = \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right)$$

$$\text{arccoth}(x) = \text{arctanh}\left(\frac{1}{x}\right) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

$$\text{arcsech}(x) = \text{arccosh}\left(\frac{1}{x}\right) = \ln\left(\frac{1}{x} + \frac{1}{x}\sqrt{1-x^2}\right)$$

$$\text{arccsch}(x) = \text{arcsinh}\left(\frac{1}{x}\right) = \ln\left(\frac{1}{x} + \frac{1}{x}\sqrt{x^2 + 1}\right)$$

Opposite Argument Formulas

$$\text{arcsinh}(-x) = -\text{arcsinh}(x)$$

$$\text{arccosh}(-x) = \text{arccosh}(x)$$

$$\text{arctanh}(-x) = -\text{arctanh}(x)$$

$$\text{arccoth}(-x) = -\text{arccoth}(x)$$

$$\text{arcsech}(-x) = \text{arcsech}(x)$$

$$\text{arccsch}(-x) = -\text{arccsch}(x)$$

Identities

$$\sinh(\text{arccosh}(x)) = \sqrt{x^2 - 1}$$

$$\cosh(\text{arcsinh}(x)) = \sqrt{x^2 + 1}$$

$$\sinh(\text{arctanh}(x)) = \frac{x}{\sqrt{1-x^2}}$$

$$\cosh(\text{arctanh}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\tanh(\text{arcsinh}(x)) = \frac{x}{\sqrt{x^2 + 1}}$$

$$\tanh(\text{arccosh}(x)) = \frac{\sqrt{x^2 - 1}}{x}$$

Sum Formulas

$$\text{arcsinh}(a) \pm \text{arcsinh}(b) = \text{arcsinh}\left(a\sqrt{b^2 + 1} \pm b\sqrt{a^2 + 1}\right)$$

$$\text{arccosh}(a) \pm \text{arccosh}(b) = \text{arccosh}\left(ab \pm \sqrt{a^2 - 1}\sqrt{b^2 - 1}\right)$$

$$\text{arctanh}(a) \pm \text{arctanh}(b) = \text{arctanh}\left(\frac{a \pm b}{1 \pm ab}\right)$$

$$\begin{aligned} \text{arcsinh}(a) \pm \text{arccosh}(b) &= \text{arccosh}\left(b\sqrt{a^2 + 1} \pm a\sqrt{b^2 - 1}\right) \\ &= \text{arcsinh}\left(ab \pm \sqrt{a^2 + 1}\sqrt{b^2 - 1}\right) \end{aligned}$$

Equations, $k \in \mathbb{Z}$

$$\sinh(x) = a \rightarrow x = \text{arcsinh}(a) + 2ik\pi$$

$$\cosh(x) = a \rightarrow x = \pm \text{arccosh}(a) + 2ik\pi$$

$$\tanh(x) = a \rightarrow x = \text{arctanh}(a) + ik\pi$$

$$\coth(x) = a \rightarrow x = \text{arccoth}(a) + ik\pi$$

$$\text{sech}(x) = a \rightarrow x = \pm \text{arcsech}(a) + 2ik\pi$$

$$\text{csch}(x) = a \rightarrow x = \pm \text{arccsch}(a) + 2ik\pi$$